Field Crop Irrigation–Multi-Objective Optimization and Sensitivity to Weather Forecast Accuracy

Theodor Bughici [1] Eran Tas [2] Erick Fredj [3] Naftali Lazarovitch [1]

[1]Ben-Gurion University of the Negev, Israel. [2]The Hebrew University of Jerusalem, Israel. [3]Lev Academic Center, School of Engineering and Computer Science, Israel.

Contact Information: Email: theodorb@post.bgu.ac.il Phone: +972 54 474 6536

Abstract

Accurate irrigation and fertigation of field crops are crucial for maximizing crop yield while avoiding overuse of water and fertilizer. Weather forecasts can predict potential evapotranspiration (ET_0) but are still far from perfect. We used a case study of sprinkler irrigated spring potatoes in Coastal Israel as a test case in order to define a minimal accuracy level of ET_0 predictions for irrigation planning. The working stages of simulation-optimization-sensitivity analysis are described in the workflow. By modeling crop irrigation based on varying forecasted ET_0 relative bias ranges as well as crop and soil parameters we were able to rank the parameters by contribution to crop-model output variance. Our main findings are (Fig. 4):

- ET_0 prediction accuracy dominates the crop model parameters when ET_0 relative bias (δ_{ET_0}) range < 5%.
- The soil n parameter dominates model output when δ_{ET_0} range < 5% for all objective functions but transpiration ($RMSE_{T_a}$).
- The max. root depth is dominating transpiration output ($RMSE_{T_a}$) when δ_{ET_0} range < 5%.

This procedure of optimization and sensitivity analysis can be extended to a wide range of case studies and help define what is an adequate weather forecast accuracy suitable to base crop irrigation upon.

Workflow

Simulation — Root water and solute uptake (HYDRUS-1D) of sprinkler irrigated potatoes in Coastal Israel. → 15,000 model runs → **Optimization** — Finding optimal irrigation management, using a multiobjective genetic algorithm (NSGA*ii*). → 11,700 model runs → **Sensitivity analysis** — Fourier amplitude sensitivity analysis (FAST) of model parameters with decreasing δ_{ET_0} ranges.

Table 1. Crop model input parameters tested for sensitivity and their ranges.

par.	description	range	par.	description	range
K_s	hydraulic conductivity (cm h^{-1})	5.7, 14.3	$Z_{r,max}$	max. root depth (cm)	40, 80
α	retention curve par. (cm^{-1})	0.04, 0.2	C_{50}	osmotic stress par. (dS m^{-1})	10.7, 14
n	retention curve par. (-)	1.94, 2.65	δ_{rain}	rain relative bias (%)	-100, 100
$h_{3,L}$	water stress par., low ET_0 (cm)	-175, -500	δ_{ET_0}	predicted ET_0 bias (%)	changing
$h_{3,H}$	water stress par., high ET_0 (cm)	-10, -175			

Forecast accuracy

Weather forecasts accuracy of ET_0 and precipitation are treated as constant **relatively biased hourly forecasts** throughout the irrigation season, where relative bias is defined as

$$\delta_x\,(\%) = \frac{x^{forecast} - x^{obs}}{x^{obs}} \times 100\% \tag{1}$$

with x being the hourly meteorological variable, forecasted or observed.

Simulation

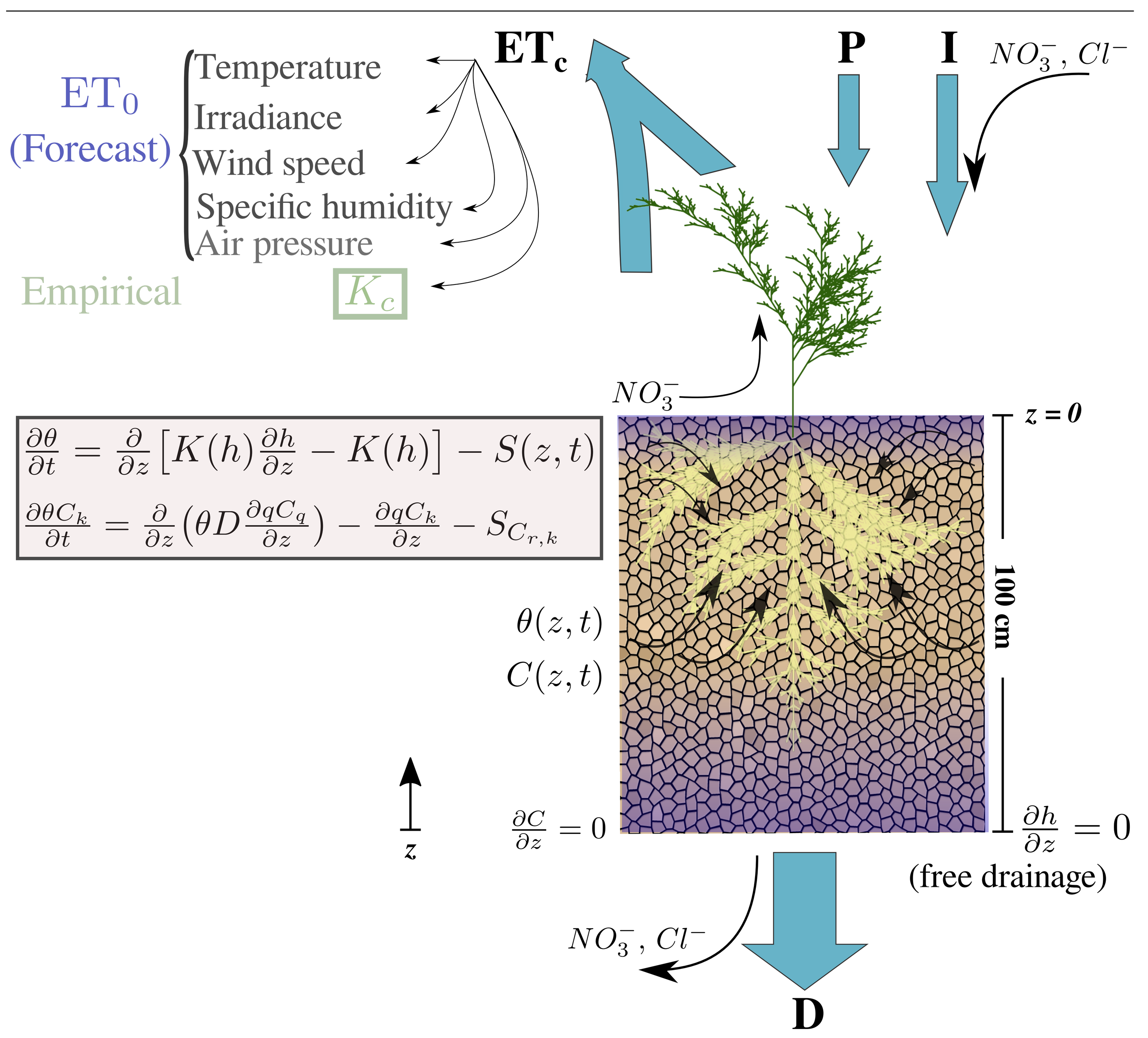

Figure 1. Crop model scheme. 1D water and solute movement and root uptake.

Optimization

The irrigation management multi-objective optimization problem is formulated as:

$$\begin{aligned} &optimize\ F(x) = (f_1(x), f_2(x), f_3(x)) \\ &subject\ to\ x \in \Omega \end{aligned} \tag{2}$$

Where Ω is the decision space. Pareto-optimal set is a solution if it is not dominated by any other solution in the decision variable space. For a given multi-objective problem, the Pareto-optimal set, $P^\star$, is defined as:

$$P^\star = \{x \in \Omega \,|\, \neg\exists x' \in \Omega \quad F(x') \preceq F(x)\} \tag{3}$$

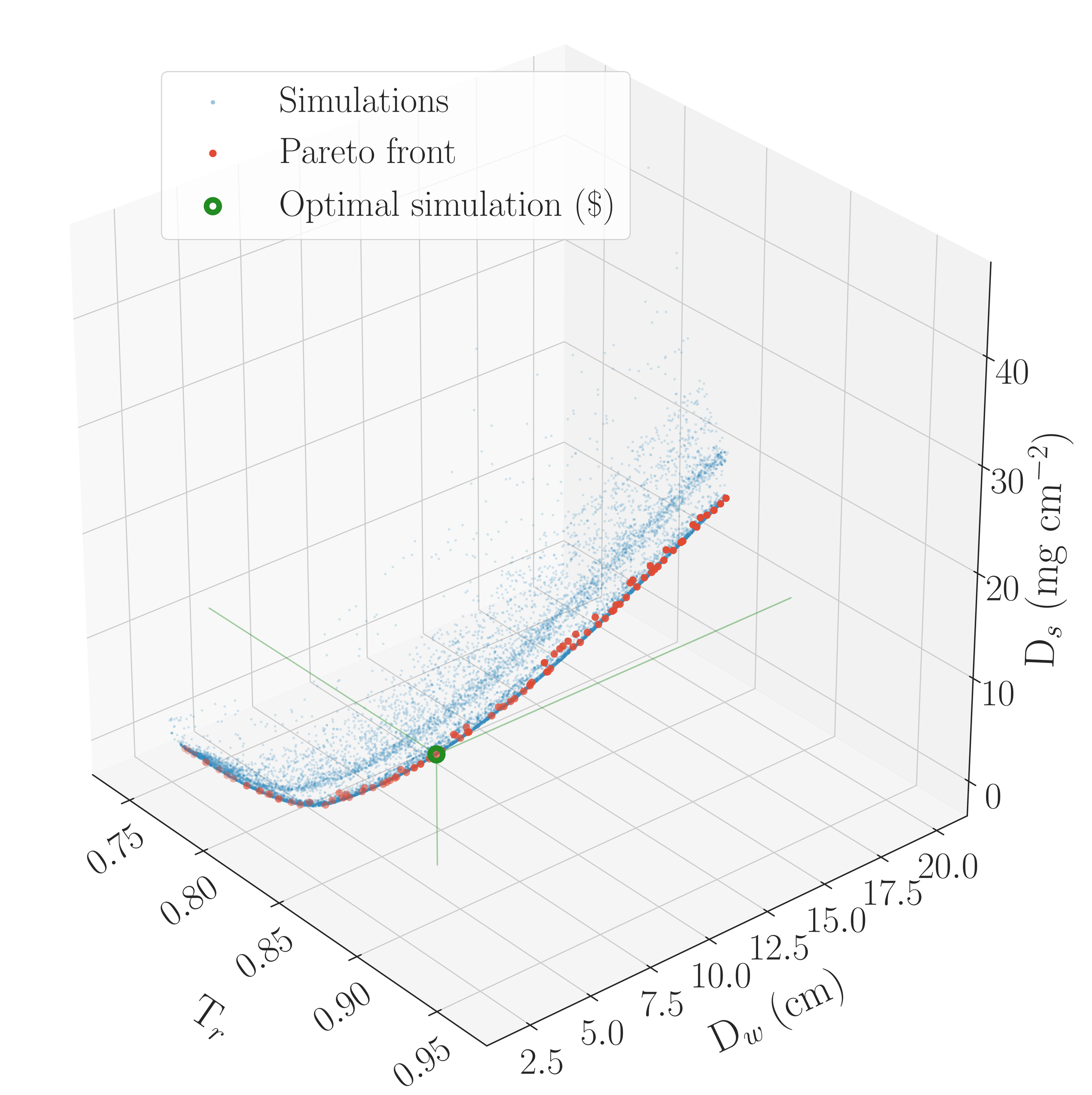

Figure 2. Objective space of the Max–Min–Min irrigation optimization problem. The objective functions are relative transpiration(T_r), water drainage (D_w) and solute drainage(D_s). The economically optimal ($) decision vector is marked in green.

Optimal irrigation management

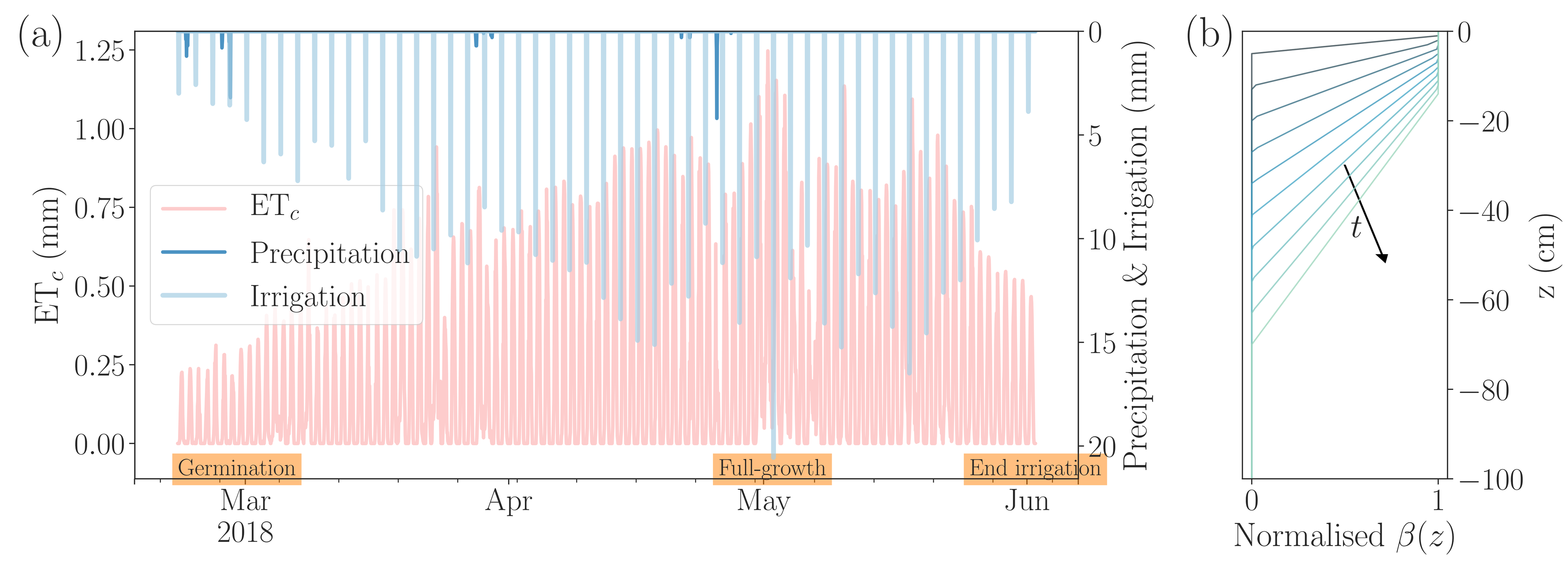

Figure 3. In and out fluxes B.C. in the field during irrigation season, managed based upon optimization results: irrigation rate = 14 $mm\,h^{-1}$, $\delta_{irrigation}$ = -1.9%, irrigation interval = 48 h with first irrigation after 3 h (a) and a description of trapezoidal root profile and growth (b) used for the sensitivity analysis simulations.

Sensitivity analysis

FAST is a variance-based global sensitivity test where the attribution of total output variance to individual model parameters and their interactions is:

$$V = \sum_{i=1}^{k} V_i + \sum_{i=1}^{k}\sum_{j>i}^{k} V_{ij} + \ldots + V_{1,2,\ldots,k} \tag{4}$$

where V is the total variance of the model output, V_i is the first-order variance for each factor x_i and V_{ij} is interactions among k factors. That allows calculation of two indices; i.e., the first-order sensitivity index corresponding to the parameter x_i: $S_i = V[E(Y|x_i)]/V(Y)$ and the total-order sensitivity index of a single parameter (index i) and the interaction of more parameters that involve index i and at least one index $j \neq i$ from 1 to k:

$$S_{T_i} = \sum S_i + \sum_{j \neq i} S_{ij} + \ldots + S_{1\ldots k} \tag{5}$$

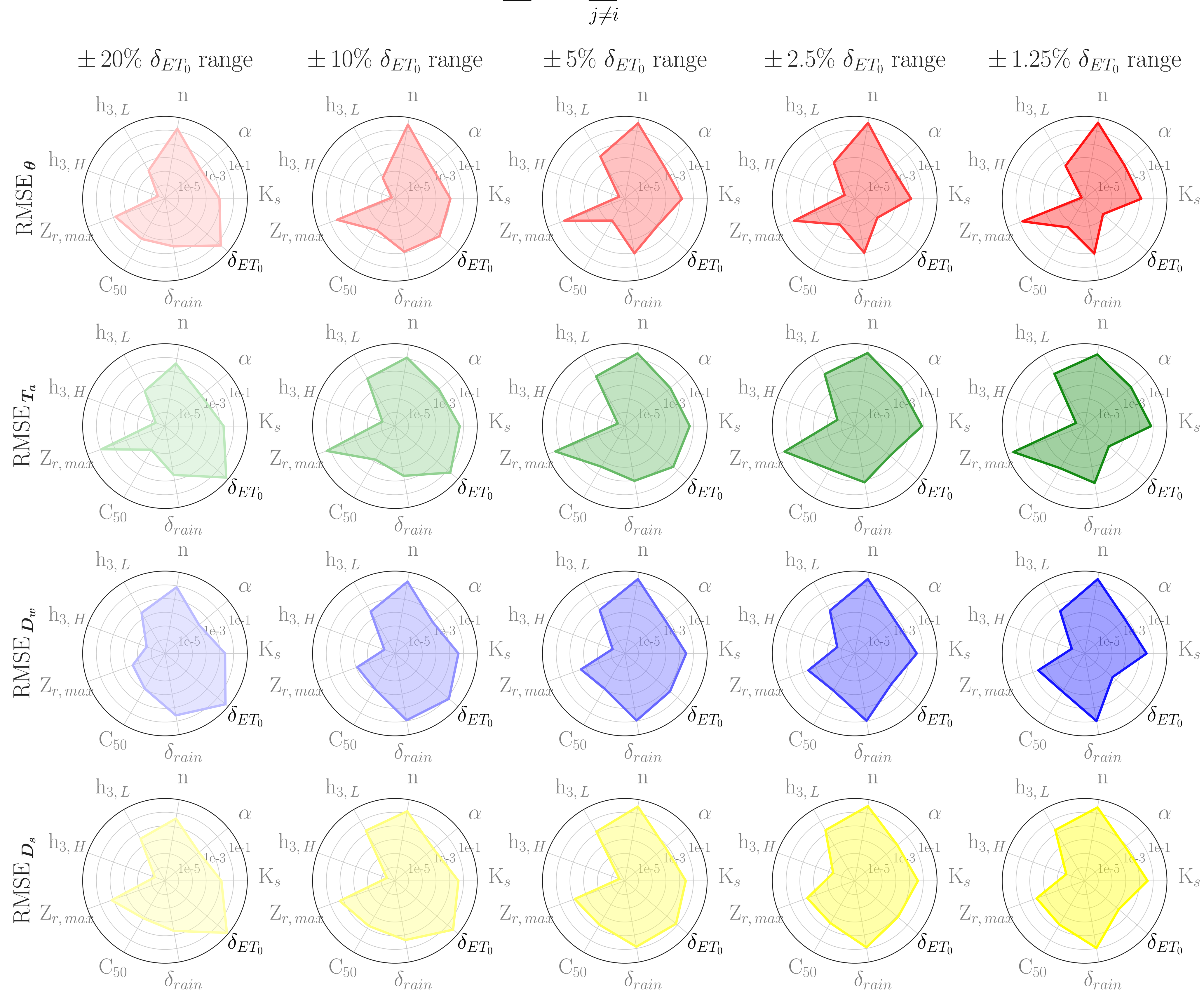

Figure 4. **Total** $\log_{10}$-sensitivity indices (S_{T_i}) of principal crop model parameters. The objective functions tested were the root mean squared error (RMSE) of the soil water content (θ) at seven depths (-10 to -70 cm), the actual transpiration (T_a), the water drainage (D_w) and solute drainage (D_s).

References

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